FORM TP 2022042

TEST CODE 01254020

MAY/JUNE 2022

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 02 – General Proficiency

2 hours 40 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of SIX questions. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. A list of formulae is provided on page 4 of this booklet.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
6. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator (non-programmable)
Geometry set

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LIST OF FORMULAE

Arithmetic Series

$$T_n = a + (n-1)d \qquad S_2 = \frac{n}{2}[2a + (n-1)d]$$

Geometric Series

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r}, -1 < r < 1 \text{ or } |r| < 1$$

Circle

$$x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x+f)^2 + (y+g)^2 = r^2$$

Vectors

$$\mathbf{v} = \frac{\hat{\mathbf{v}}}{|\mathbf{v}|} \qquad \cos\theta = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}| \bullet |\mathbf{b}|} \qquad |\mathbf{v}| = \sqrt{(x^2 + y^2)} \text{ where } \mathbf{v} = x\mathbf{i} + y\mathbf{j}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Kinematics

$$v = u + at \qquad v^2 = u^2 + 2as \qquad s = ut + \frac{1}{2}at^2 \qquad v = \frac{dx}{dt} = \dot{x}$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$$

Calculus

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1} \qquad \frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax \qquad s = \int v\,dt \qquad v = \int a\,dt$$

Statistics

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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SECTION I

ALGEBRA, SEQUENCES AND SERIES

ALL working must be clearly shown.

1. (a) Consider the quadratic equation $qx^2 - (4p)x + pq^2 = 0$, where p and q are both **positive** integers.

(i) Express the sum AND product of the roots of the equation in terms of p and q.

(3 marks)

(ii) Determine the value for q such that the sum of the roots is equal to the product of the roots.

(2 marks)

(iii) If the sum of the roots of the equation is 20, use your answer from (a) (ii) to determine a value for p.

(1 mark)

(iv) Hence, express the given quadratic equation in terms of its numerical coefficients.

(1 mark)

(b) A series is given by

$$25 - 5 + 1 - \frac{1}{5} + \frac{1}{25} \ldots$$

(i) Show that the series is geometric.

(2 marks)

(ii) Calculate the sum to infinity of the series, giving the answer to **2 decimal places.**

(2 marks)

(c) A recent university graduate was offered a starting salary of $720 000 for the first year, with increases of $5000 at the start of every year thereafter. Determine the number of years (to the nearest whole number) that it would take for her annual salary to be 20% greater than her salary in the first year.

(4 marks)

Total 15 marks

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2. (a) When the polynomial expression $2x^3 - 3x^2 - cx + d$ is divided by $(x + 1)$ and $(x - 2)$, the same remainder of 64 is obatined.

Determine the value of c and d.

(4 marks)

(b) Show that the expression $\frac{\sqrt{25}}{\sqrt{45}}$ is the same as $\frac{\sqrt{5}}{3}$.

(2 marks)

(c) (i) Given $g(x) = 6x^2 + 12x - 18$, express $g(x)$ in the form $a(x + h)^2 + k$.

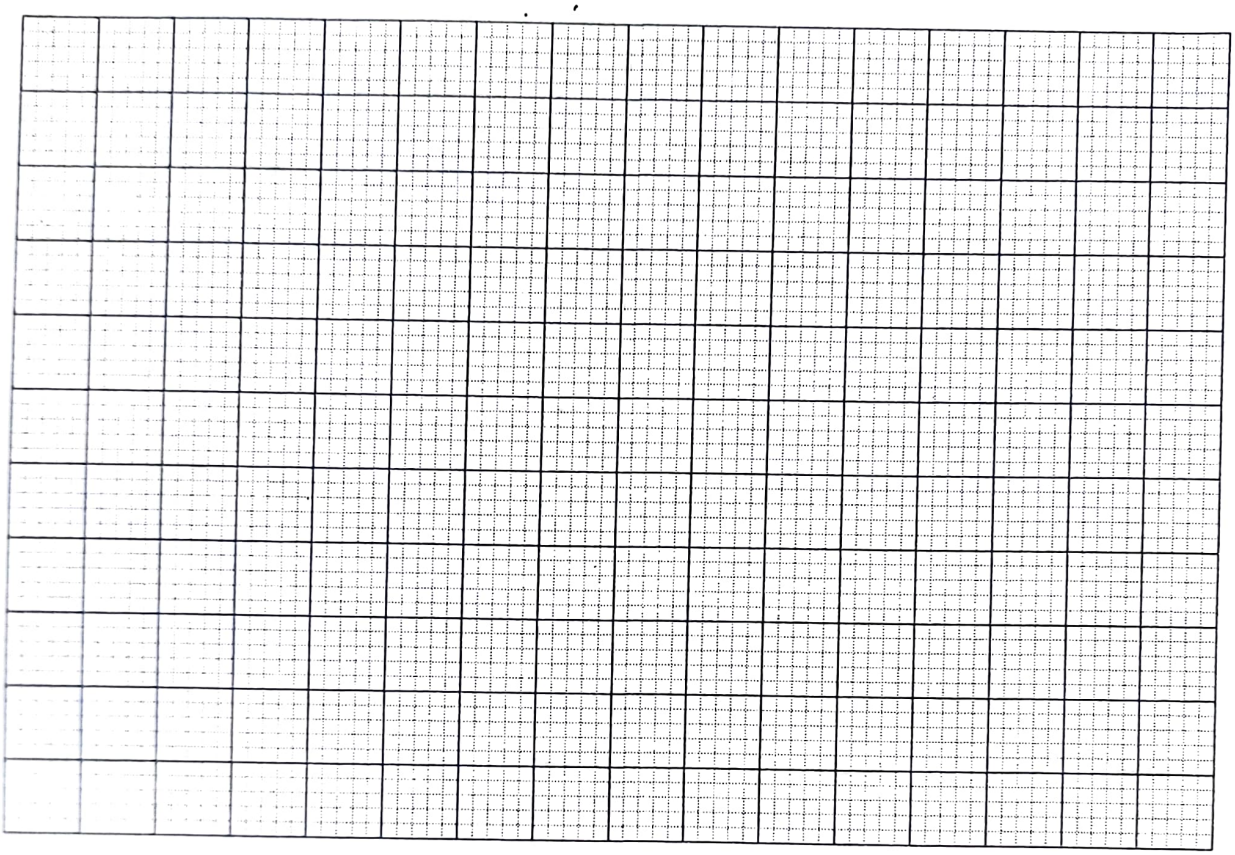
(3 marks)

(ii) Using the expression derived in (c) (i), determine the roots of $g(x)$.

(3 marks)

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(iii) Hence, sketch the graph of $g(x)$ on the following grid.



(3 marks)

Total 15 marks

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SECTION II

COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

ALL working must be clearly shown.

3. (a) The equation of a circle is $x^2 + y^2 + 4x - 8y + 10 = 0$.

(i) Determine the coordinates of its centre AND the length of the radius of the circle.

(3 marks)

(ii) Determine the equation of the tangent to the circle at the point P (–5, 5).

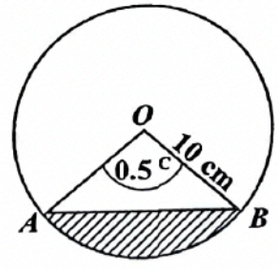
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(b) The vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are such that $\overrightarrow{OX} = 4\mathbf{i} + \mathbf{j}$ and $OY = \mathbf{i} - 4\mathbf{j}$. Show that the vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are perpendicular.

(5 marks)

(c) The diagram below, **not drawn to scale**, shows a chord AB which subtends an angle of 0.5^c (0.5 radians) at the centre, O, of a circle of radius 10 cm. Given that the area of triangle $AOB = \frac{1}{2} r^2 \sin\theta$, calculate the area of the shaded region.



(3 marks)

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(d) (i) Show that $\cos 2\theta = 2\cos^2\theta - 1$.

(3 marks)

(ii) Hence, solve the equation $\cos 2\theta + \cos\theta + 1 = 0$ for $0 < \theta < 2\pi$.

(3 marks)

Total 20 marks

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SECTION III

INTRODUCTORY CALCULUS

ALL working must be clearly shown.

4. (a) Given that $f(x) = x(5 - x)^2$, determine $f''(x)$.

(3 marks)

(b) Differentiate EACH of the following expressions with respect to x, simplifying your answer where possible.

(i) $2 \sin 3x + \cos x$

(3 marks)

(ii) $(1 + 2x)^3 (x + 2)$

(3 marks)

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(c) (i) Determine the stationary points on the curve $y = x^3 - 4x^2 + 4x$.

(4 marks)

(ii) Providing details, determine the nature of EACH stationary point in (c) (i).

(2 marks)

Total 15 marks

5. (a) Determine $\int (4 \cos \theta - 6 \sin \theta)\, d\theta$.

(2 marks)

(b) Evaluate $\int_1^2 (3-x)^2\, dx$.

(3 marks)

(c) Determine the area of the region bounded by the curve $y = 5 + 5x - x^2$, the x-axis, the y-axis and the line $x = 2$.

(4 marks)

(d) A particle moves in a straight line so that t seconds after passing through a fixed point, O, its acceleration, a, is given by $a = (3t - 1)$ m s^{-2}. When $t = 2$, the particle has a velocity, v, of 4 m s^{-1}, and a displacement of 6 m from O.

Determine the velocity when $t = 4$.

(6 marks)

Total 15 marks

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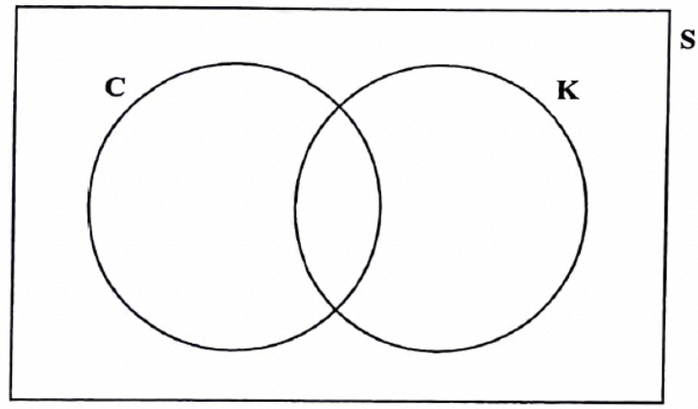
SECTION IV

PROBABILITY AND STATISTICS

ALL working must be clearly shown.

6. (a) At a school canteen, 80% of the students (S) purchase chips (C) and 55% purchase chicken (K). Of the students who purchase chicken, 11% do not purchase chips.

(i) Complete the following Venn diagram to illustrate this information.



(4 marks)

(ii) Determine the probability that a student chosen at random purchases ONLY chicken or ONLY chips.

(1 mark)

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(b) The probabilities of the occurrence of two events, A and B, are given by $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{7}{10}$. Determine

(i) $P(A \cap B)$

(3 marks)

(ii) $P(A|B)$.

(3 marks)

(c) State, with a reason, whether Events A and B are independent.

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(1 mark)

(d) The following table shows the marks obtained by 27 students in a Mathematics test.

30	35	39	42	45	45	52	59	61
61	65	69	70	71	75	77	79	81
83	83	85	87	89	90	90	95	98

(i) Construct a stem-and-leaf diagram to display this data.

(4 marks)

(ii) State ONE advantage of using a stem-and-leaf diagram to display the data.

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(1 mark)

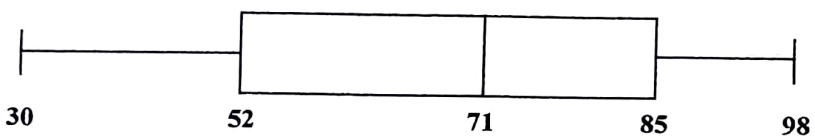
(iii) State the range of values of the marks obtained by the students.

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(1 mark)

(iv) The data are displayed in the following box-and-whisker plot.



State TWO distinct observations about the data as seen in the box-and-whisker plot.

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(2 marks)

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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